Louisiana Highway Research

QUALITY CONTROL ANALYSIS

PART II

SOIL & AGGREGATE BASE COURSE

Louisiana Department of Highways – Research & Development Section

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by

S. C. SHAH
Assistant Research Engineer

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SYNOPSIS

This is the second of the three reports on the quality control analysis of highway construction materials.

It deals with the statistical evaluation of results from several construction projects to determine the basic pattern of variability with respect to certain base course characteristics. On the basis of this variability, numerical limits have been established using statistical quality control techniques.

The analysis indicated (1) that the frequency distribution of historical data for most of the characteristics tend to follow normal distribution; (2) that the variability for compaction and thickness is considerably different for different contractors; (3) that this variability for compaction is more pronounced for cement stabilized aggregate base course than for stabilized soil cement course; (4) furthermore, that for raw or unstabilized aggregate base course, the variability is less than that for stabilized base course.

INTRODUCTION

This report is concerned with the application of Statistical Quality Control technique to writing some of the highway materials specifications.

In its simplest form, quality refers to the quality of conformance; that is, does the end product conform to some preset standard? If it does, then the product is said to have quality built into it.

Secondly, control is the mechanism whose primary function is the prevention of defects.

Lastly, Statistical Quality Control is a systematic procedure which involves random collection of representative data and analysis of this to determine whether all or any part of such data is drawn from within or without a chance cause system. Such analysis results in conclusions based on predetermined levels of confidence and probability. In the industry application of such technique has provided an important tool to both the producer and the consumer for acceptance and/or rejection of the manufactured products. Highway construction need not be considered different from any manufacturing industry where there is a Producer identified as a Contractor; there is a Consumer which is the State Highway Department; and finally, the manufactured product which could be the finished roadway or any related structure. Realizing this, highway engineers have directed their attention to application of such techniques in order to:

1) Determine the basic pattern of variability concerning each material characteristic.

2) Determine process capabilities.

3) Aid in the establishment of realistic specifications.

4) Aid in better over-all producer-consumer relationships.

While it is fundamentally true that the only excuse for specifications in any regard is to outline some necessary phase of performance, it is also true that these specifications be based on facts. Whenever specifications go into the realm of conjecture, then difficulty is made certain, costs pyramid, and waste is inevitable.

Louisiana Department of Highways has embarked on a total quality control program. The contents contained herein constitute the second phase of this program and specifically involves soil and aggregate base course characteristics.

SCOPE

Total quality control program at the Louisiana Department of Highways basically involves analysis of certain problematic highway construction material characteristics for variability. The program was initiated in late 1963 in cooperation with the Bureau of Public Roads and is broken down into separate phases. Findings of the first phase is reported in reference (1). The contents of this report represent the second phase of the study with emphasis on the following base course characteristics:

(1) Aggregate Base Course

  a. Density
  b. Gradation of sand-clay-gravel
  c. Thickness

(2) Stabilized Sand-Clay-Gravel and Sand-Shell Base Course

  a. Density
  b. Thickness
  c. Width

(3) Stabilized Soil-Cement Base Course

  a. Density
  b. Thickness
  c. Width
  d. Moisture content for mixing and compaction

Realizing that persons (particularly those within the Louisiana Department of Highways) without statistical background would be at a loss in deriving any anticipated benefit from the contents of this report, an attempt has been made whenever deemed necessary to go over the fundamentals pertaining to the topic.

SAMPLING AND TESTING METHODS

Unless otherwise mentioned, the sampling methods are according to:

1. LDH Designation: S 104-64 - Standard Method of Sampling Aggregate for Base and Surface Courses
2. LDH Designation: S 404-64 - Standard Method for Sampling of Subgrade for Application of Soil-Cement, Soil Lime, Soil Asphalt, or other types of Stabilized Bases

The test methods are according to:

1. LDH Designation: TR 401 - Method of Test for the Determination of In-Place Density
2. AASHO Designation: T 27 - Method of Test for Mechanical Analysis of Fine and Coarse Aggregates

The currently used specifications by the Department for job conformance are indicated in Table I.

TABLE I

REQUIREMENT FOR PHYSICAL PROPERTIES OF COMPACTED BASE COURSE

Physical Characteristic	Allowable Tolerance and/or Specification Requirement	Frequency and/or No. of Measurements
% of Maximum Density		
(1) Cement Stabilized	95%	n=5,one every 500 linear feet
(2) Aggregate (raw)	100%	n=3,one every 500 linear feet
Plan Thickness		
(1) 6" and under	-1/2" and +1"	n=3
(2) 6" to 8"	-3/4" and +1 1/4"	
(3) 8" and over	-1" and + 1 1/2"	
Width	±3" on either side of center line	n=3
Gradation	Per Cent Passing	
	3/4" #4 #40 #200	
(1) Sand-Clay-Gravel "A"	75-95 40-60 20-45 10-20	Not Specified
(2) Sand-Clay-Gravel "B"	- 50-75 20-50 12-25	Not Specified

OUTLINE OF WORK

Collection of Data

Quality control program necessitates the gathering of vast amounts of data. Furthermore, it is almost implicit that these data are unbiased and a religious adherence to random selection of samples is usually necessary to ensure this lack of bias. To accomplish this, a specially designed sampling plan (using random number tables) would have been an ideal approach in obtaining data necessary for development of statistical parameters for various characteristics. However, aside from selecting jobs under construction in various parts of the State, this controlled field experiment would have also involved considerable amount of time and personnel. Therefore, it was decided that much of the information which resides, untouched, in long rows of filing cabinets could be used to advantage without resorting to any additional sampling and testing.

After careful selection of projects on the basis of their geographic locations, quality of workmanship and type of material, accumulation of data was accomplished using the following sources:

1. Daily Inspection Reports
2. Laboratory Reports
3. Record Test Reports

In adopting such an approach, it is assumed that:

1. The currently used sampling and testing techniques are sound.
2. The construction techniques are adequate.
3. The data are representative and free of any bias.

A limitation to the last one is that the data may not have come from a randomly selected sample and there may be some bias in reporting. In statistical evaluation, an out-of-specification result is just as important as the one within the specification limits for then only can a true estimate of variability be ascertained. However, a statistical reliability check was made on the historical data to ascertain the last assumption.

Field Work

Controlled field experiment (using random number tables) was also conducted on four separate projects selected on the basis of their geographic location. All of these projects involved construction of cement stabilized base course. Samples were obtained to evaluate the following characteristics:

1. Density - one at each of the 30 randomly selected locations.
2. Width - same as above.
3. Thickness - three measurements; one at center line, and one each to the right and left of center line at each of the 30 locations.
4. Moisture content - two replicate determinations at 50 random locations.

Data Processing

The statistical parameters were calculated using the Department's IBM 1620 computer and Fortran II Compiler System.

Test Results

Summary of statistical results on various base course characteristics is presented in the Appendix in Tables VII through XII.

CHOICE OF ESTIMATORS OF PARAMETERS

Since it is impossible to obtain a true mean and standard deviation of the population, it is necessary to make good estimates of these parameters; that specification limits be closely related to the actual behavior of the process is one of the many reasons for these parameters to be unbiased and efficient.

The mean, $\overline{X}$, is a measure of central tendency of a group of measurements. Mathematically:

$$\overline{X} = \frac{\Sigma X_i}{n} \qquad \text{where ,}$$

X_i = individual observations,

and n = number of observations in a group.

The standard deviation, σ (sigma) , is a measure of the dispersion of the measurements from their mean. The mathematical definition is:

$$\sigma = \sqrt{\frac{\Sigma(X_i - \overline{X})^2}{n-1}} \qquad \text{where ,}$$

X_i , $\overline{X}$ and n are as above.

The variance, σ^2 , is the square of the standard deviation.

The standard error, $\sigma_{\overline{X}}$, is the standard deviation of the mean of several samples and is estimated by:

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$$

ANALYSIS OF DATA

Frequency Distribution

One of the most commonly used methods of describing pictorially variations of individual observations from within a sample is by means of frequency distribution. In examining data of such type (or any other type) it will be found that the individual observations group themselves about the central value so that there are roughly equal numbers on either side of this central value and small divergencies from this central value occur more frequently than large ones. When this happens, the resulting curve assumes what is termed a Gaussian or Normal Distribution which has a symmetrical bell shape. This is one of the most important distributions in statistics and forms the basis for subsequent analysis of the present data. Its use is the same as that of any other distribution curve; the relative frequency with which a variable will take on values between two points is the area under the curve between the two points on the horizontal axis.

If, instead of plotting individual observations, means of several sample units were plotted, then the resulting distribution would be much narrower. These relationships are shown graphically in Figure 1. Table II shows typical frequency distribution data for base compaction on one of the projects. Graphical representation of this distribution is shown in Figure 2. Figures 3 and 4 represent similar relationship for aggregate gradation. In spite of the limited number of observations, the curves do suggest a normal distribution as indicated by the bell shape. Departure from this tendency can be attributed to sampling and testing errors. The lower half of these figures, which is a cumulative frequency distribution curve on normal probability paper, gives empirical evidence of the normality assumption and further indicates that the data can be considered amenable to further treatment by established statistical procedures for writing realistic specifications.

Some of the important characteristics of such normal probability curves can be used to represent the accumulated data on different characteristics. If the horizontal axis of this curve is represented by the normal deviate (which is the number of standard deviations of the measurements above or below the mean value), then the area under the normal curve between any two values of the normal deviate (t_1 and t_2) gives the probability that an observation from the population will have a value between t_1 and t_2.

Table III shows some of the selected critical values for this normal distribution.

The table illustrates that for a normally distributed data, 95.45% of the results will be within plus or minus 2σ or that approximately 4.55% of the results will be outside the range of $\pm 2\sigma$. Likewise, 99.73% will be within $\pm 3\sigma$ and corresponding .27% outside this range. Thus, knowing the true value of the mean and standard deviation, one can set up limits within which a predetermined proportion of observations shall be included. Use of this table will become evident when it is applied to practical examples later in the report.

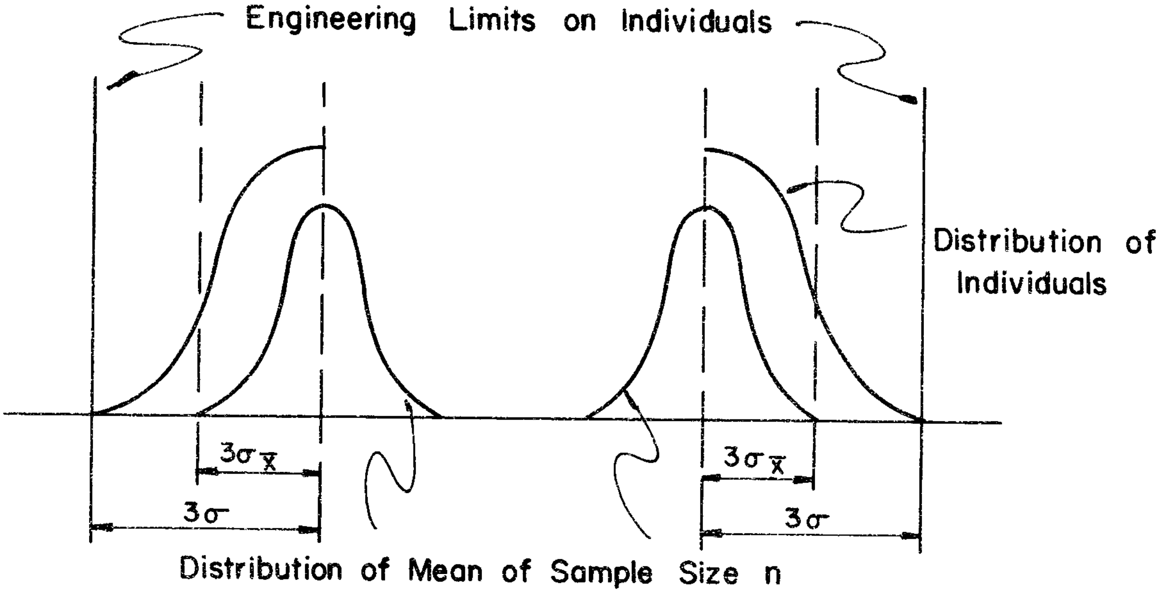


Figure 1: Relationship Between Distribution of Individual Observations and Means of Several Sample Units.

TABLE II

TYPICAL FREQUENCY DISTRIBTUION DATA FOR BASE COURSE COMPACTION

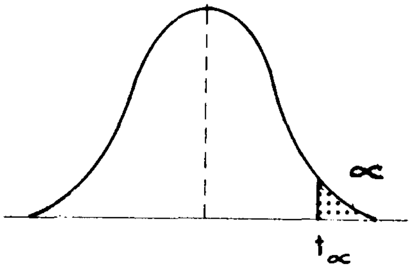
Class Interval 1.5%

x	f	fx	x^2	$f(x)^2$	Σf	Σf, %
-6	2	-12	36	72	2	.8
-5	2	-10	25	50	4	1.5
-4	3	-12	16	48	7	2.6
-3	9	-27	9	81	16	6.6
-2	29	-58	4	116	45	17.0
-1	41	-41	1	41	86	32.5
0	51	0	0	0	137	51.7
1	46	46	1	46	183	69.1
2	43	86	4	172	226	85.3
3	19	57	9	171	245	92.4
4	15	60	16	240	260	98.1
5	5	25	25	125	265	100.0

TABLE III

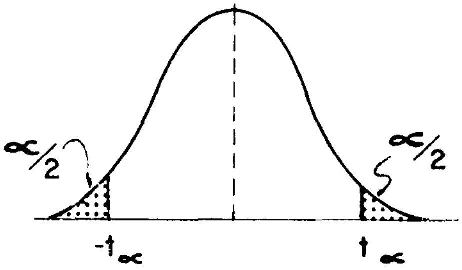
SELECTED VALUES FOR THE NORMAL DISTRIBUTION

One Tail Critical Values



α or β	t_α or t_β
.10	1.282
.05	1.645
.0455	1.690
.025	1.960
.02	2.054
.01	2.326
.005	2.576
.0027	2.782
.002	2.878
.001	3.090

Two Tail Critical Values



α or β	t_α or t_β
.10	1.645
.05	1.960
.0455	2.000
.025	2.241
.02	2.326
.01	2.576
.005	2.807
.0027	3.000
.002	3.090
.001	3.291

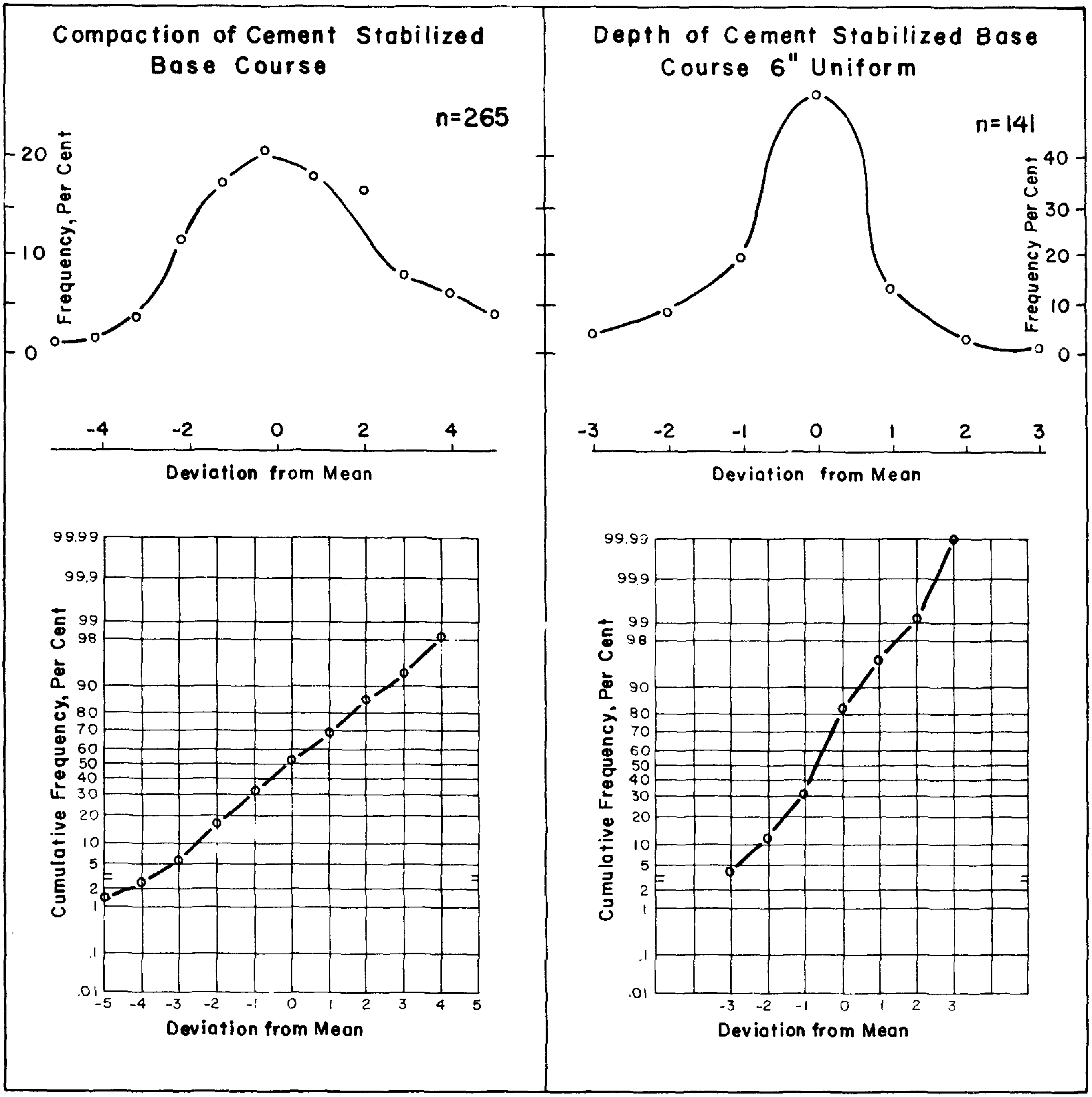


Figure 2: Frequency Distributions for Per Cent Compaction and Thickness of Cement Stabilized Base Course Deviations.

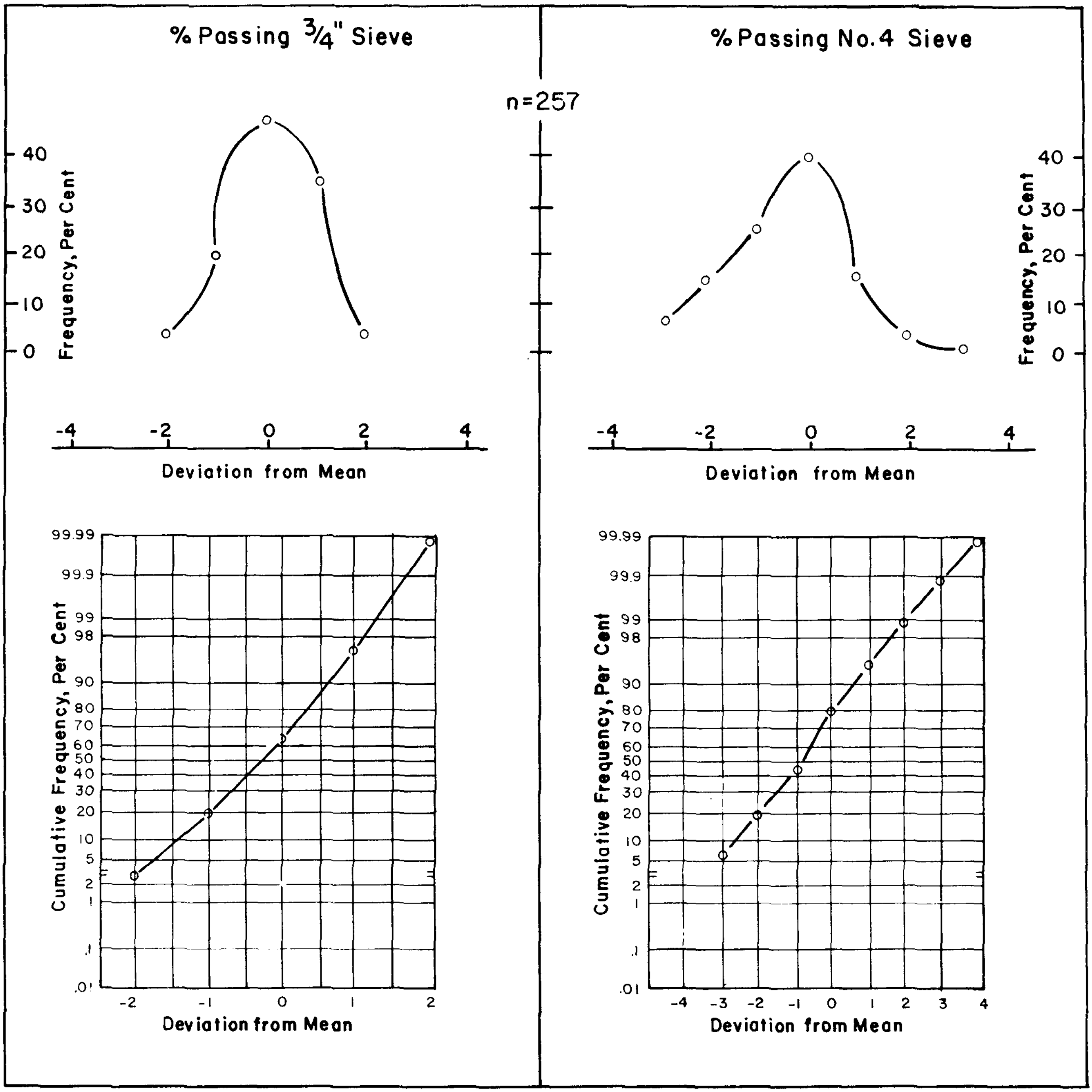


Figure 3: *Frequency Distributions for 3/4" and No. 4 Size Aggregate Deviations (Data from Project A for Sand-Clay-Gravel Grade "A").*

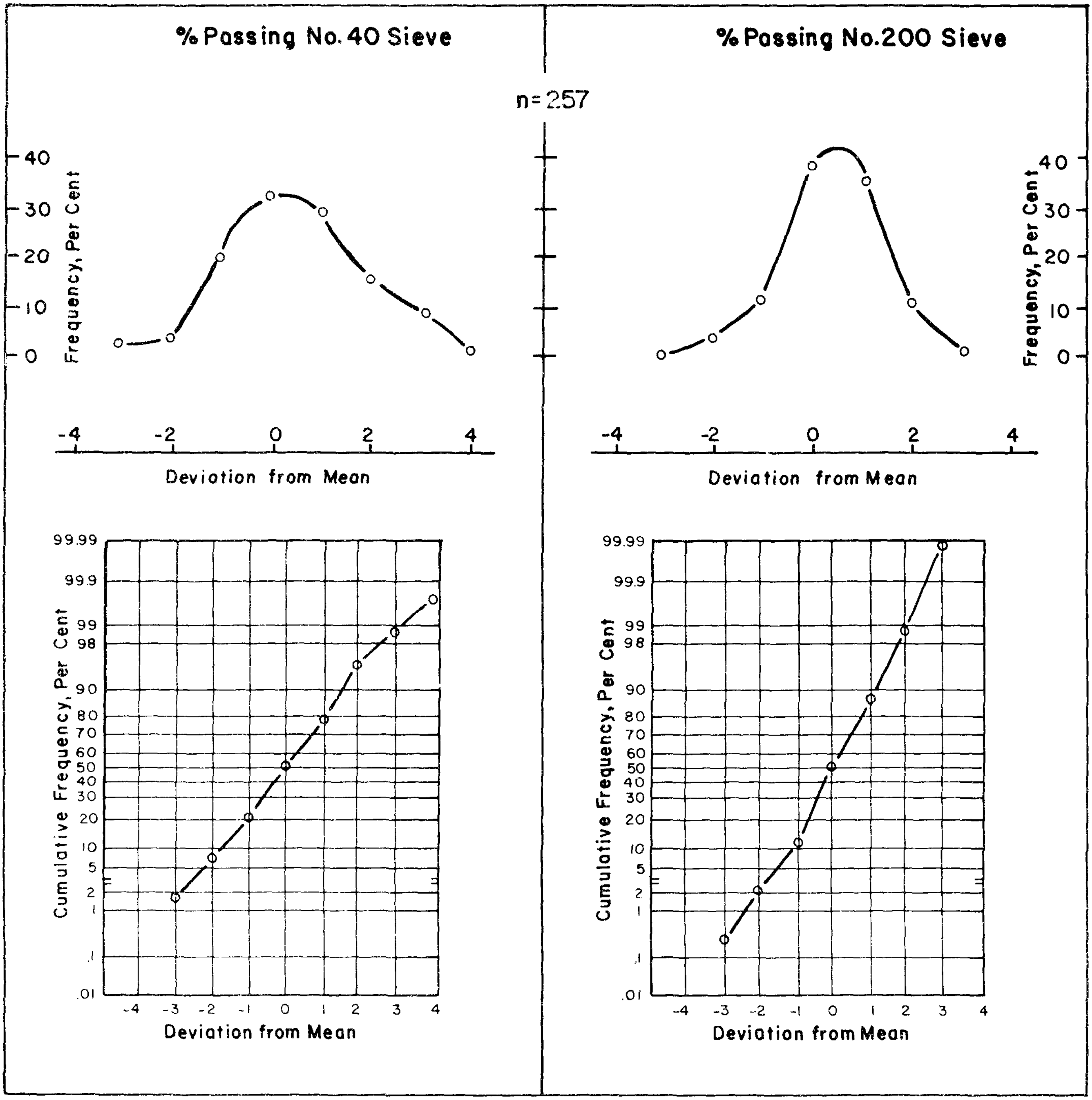


Figure 4: Frequency Distributions for No. 40 and No. 200 Size Aggregate Deviations (Data from Project A for Sand-Clay-Gravel Grade "A").

RELATIONSHIP BETWEEN SPECIFICATIONS AND STATISTICAL PARAMETERS

Figure 5A symbolizes a relationship between specification tolerance limits and statistical parameters using an idealized normal distribution curve. It indicates great variation with respect to the tolerance limits. This situation is untenable and three solutions are available to modify this situation.

1. Remove the fringe lying outside each tolerance by measuring each and every item which is undoubtedly a costly procedure.

2. Find a new and better method to measure the characteristic (involving research and delay).

3. Revise the limits by making it wider. There is no point in making the specifications so tight they can not be enforced.

Figure 5B shows a situation where the curve just clears the inside limits. At first, this might seem to be perfect. However, on second thought, there does not seem to be any allowance for operating tolerance and the dotted line shows how the measurements would be outside the limits with only a slight shift in the mean.

The most comfortable situation is illustrated in Figure 5C, where some leeway for sampling, testing, or material variation is allowed. Under this condition, adequate conformance with specification tolerance can be expected.

The above was an idealized, hypothetical case. Such relationships for some of the results obtained in this study are illustrated in Figures 6A through 6I for base course compaction. Statistical information is given in Table VII. These figures illustrate that in order for all test results to conform to the minimum specification of 95%, the process (construction) need to be maintained at 95 + 3.09 (σ), 3.09 being the value of the normal deviate in Table III and σ being the standard deviation of the characteristic. Conversely, for all results to be acceptable, the specification need to be changed to $\overline{X}$ - 3.09 (σ). For example, Contractor A, whose variability σ was 1.96% and $\overline{X}$ = 98.71, the lower specification limit should be 98.71 - 3.09 (1.96) or equal to 92.83% for 100% conformance. On the other hand, if we are ready to relinquish, say 2.5% the results, then the specification may be set at 98.71 - 1.96 (1.96) or 94.79%. Similarly, for Contractor B who had larger value of sigma and $\overline{X}$, the lower limit should be 90.07% for 100% acceptance and 93.04% for 97.5% acceptance.

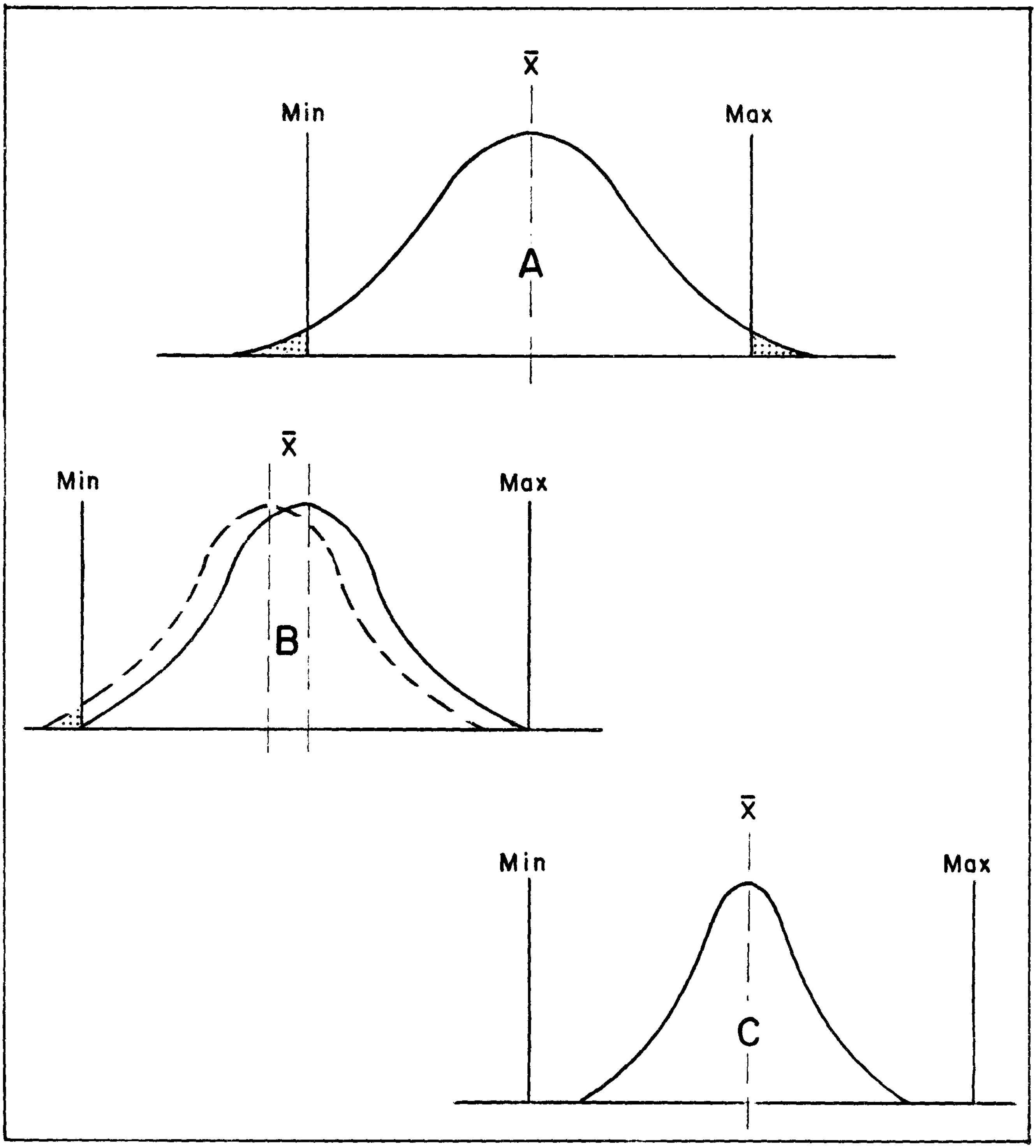


Figure 5: Some Distributional Aspects of Specifications.

Similar reasoning can be applied to other contractors (projects) and is indicated in the figures.

Most of the present day specifications are so worded as to imply all or 100% of the product to conform to specifications. Statistically, acceptance of a lot (section of a roadway, stockpile of material, etc.) on the basis of solitary measurement (n=1) is irrelevent. Furthermore, the risk of accepting a bad lot also becomes great. For this reason, it is believed that the compliance on means would be more meaningful than on individuals. However, one question to answer is the size of the sample since specifications can be written for a variety of sample sizes and these will be different for each sample size.

To avoid confusion, the term "sample" should be defined. In highway terminology, this implies a single measurement such as gradation determination on a sample of sand-clay-gravel. In statistical parlance, a sample is a composite of several units of measurements. For instance, if five roadway specimens were tested for density measurement, we are generally inclined to say that five samples were taken. However, in statistical sense it would be identified as a sample of size five. Thus, a sample may consist of one observation or one million observations.

To illustrate how specification limits change for different sample size, consider the relationship

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$ which is for the standard deviation

of the mean of several samples.

For $n=4$, $\sigma_{\bar{x}} = \frac{1.96}{\sqrt{n}} = 1.0$ (approx.) for Project A, and

for $n=9$,

$$\sigma_{\bar{x}} = \frac{1.96}{\sqrt{9}} = .67$$

If 100% compliance is required for the mean, then, for $n=4$, the limit would be 98% and for $n=9$, this limit would be 99%. This is clearly illustrated in Figure 7. The point that is being made here is that limits should be based on specified sample size. Larger sample size would require lower tolerance limits and vice versa.

Table VII, which gives values for variability for different projects on the basis of historical data, brings up the question as to which particular σ should be used in setting tolerance limits on the characteristic. Contractor A was able to keep his variability quite low compared to the rest of them. This means that there could be different acceptance levels for different contractors. For instance, if

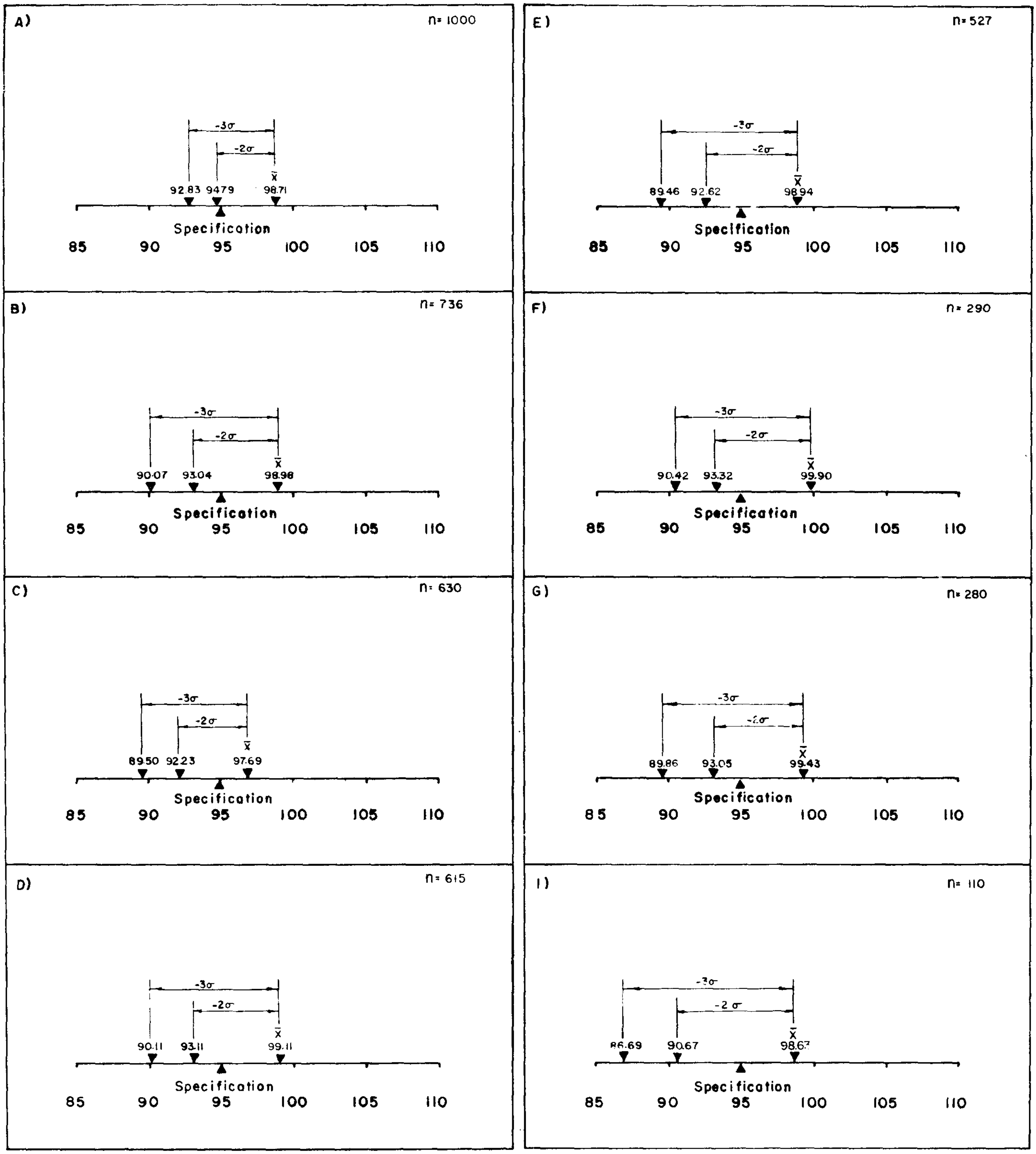


Figure 6: Relationships Between Specifications and Statistical Parameters for Stabilized Base Course Compaction.

Contractor A and Contractor I both produce approximately the same mean density of say 96.5% (based on n=4) with $\sigma_A = 1.96$ and $\sigma_I = 3.98$, then Contractor A's product should be accepted and that of Contractor I turned down. This makes it necessary for the value of sigma to be realistic and representative. This sigma should account for variations due to Men, Machinery, and Materials. Generally, several sample variances when pooled or averaged should give an estimate of total population variance. The same can be said of the population mean. Table XII represents summary of statistical results on some of the base course characteristics on the basis of controlled field experiments. The extent of variability as expressed by σ is from 2.87% for Contractor C to 4.64% for Contractor D. Contractor A's mean was so low and the variability so high that as many as 50% of the individual results failed to meet the requirement of 95% compaction. On the other hand, Contractor C, although successful in keeping his variability low, could not maintain high enough mean which resulted in 45% of the results to fall outside the minimum requirement.

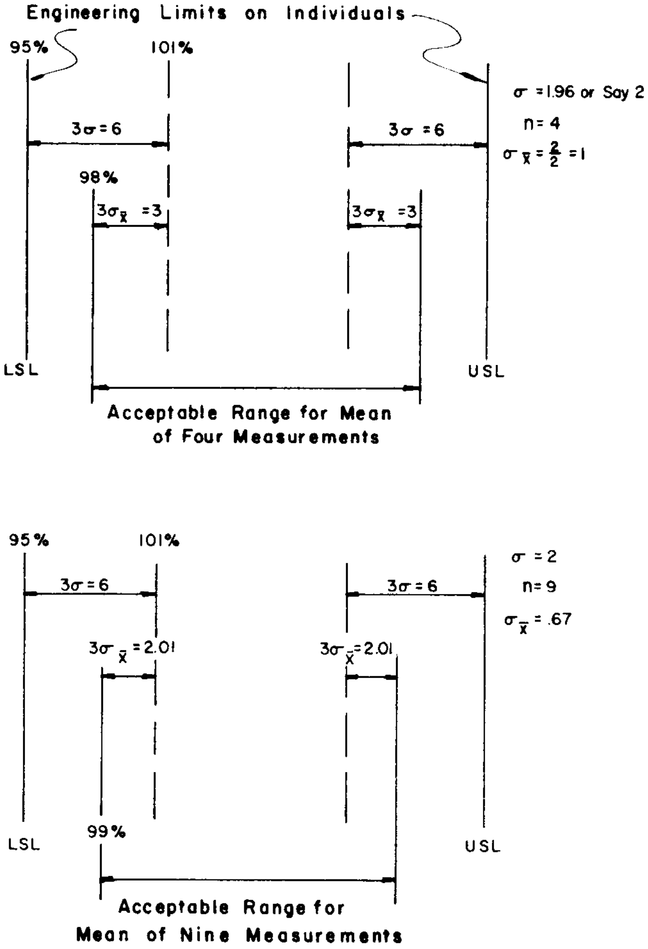


Figure 7: Effect of Sample Size on the Tolerance Limits.

SAMPLING PLAN FOR ACCEPTANCE USING VARIABLES

The acceptance function of any inspection must be coupled with well adopted sampling plan. Such a plan should specify the following:

1. Size of the sample.
2. The critical value or values of the variable for lot acceptance.
3. The probability of accepting bad products and rejecting good ones.

Various sampling plans are available and each has its advantages and disadvantages. In the industry, the principle of attributes inspection is widely used. In this, an item is classified either as defective or non-defective or count the number of defects in it. In variable inspection, the characteristic in question is measured along a continuous scale in terms of pounds per cubic foot, inches, psi, seconds, etc. Such a plan yields more information regarding the quality of the lot than does attributes sampling. Another practical advantage of using the variable inspection plan is the reduced sample sizes required for specified degrees of protection.

The use of variable sampling plan requires rather strong assumption about the nature of the distribution of the quality characteristic under consideration, viz., that it be normal. The frequency distribution of many measurements is roughly normal, and hence from practical point of view, this assumption is considered valid.

One-Way Protection on Means

Basically, the plan calls for determination of sample size n and acceptance tolerance E and operates as follows:

1. Select a random sample of size n from the lot.

2. Find $\overline{X}$, the mean of this lot.

3. If $\overline{X}$ is greater than or equal to some value K, we accept the lot; otherwise, reject it or take corrective action.

To any sampling plan, we are required to associate what is called the producer's and consumer's risk. In acceptance sampling, there is always a chance that one may erroneously reject a good lot which would be a sacrifice for the producer or contractor. This is the producer's risk, (α). On the other hand, one may accept a bad lot erroneously which would be a risk for the consumer or the State, (β). What can be considered satisfactory risks depends on the criticality of the variable and the economics of sampling and testing. However, if the variable is considered critical enough as to affect the successful performance of the end product, then the risks should be adopted accordingly. It is believed that for a major characteristic such as compaction of base course, the producer's risk can be set at .02 and the consumer's risk at .05. $\alpha = .02$ means that the probability of rejecting lots of acceptable mean quality is .02. Likewise, $\beta = .05$ means that 5% of the time bad lots would be accepted if offered by the contractor. This is shown graphically in Figure 8.

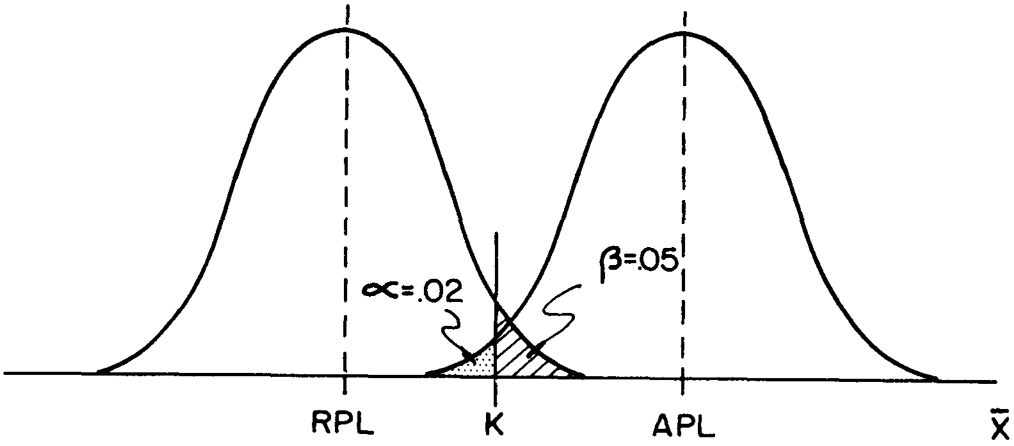


Figure 8: Distributions of $\bar{X}$ for Acceptable and Rejectable Process Levels and the Corresponding Risks (One-way Protection on Means).

To determine the value of n, the sample size, the following equations are needed. Since our test is a test on means, we shall be working with the distribution of means.

Since we have agreed to reject good lots 2% of the time

$$\frac{K-APL}{\sigma/\sqrt{n}} = -2.054 \qquad \ldots\ldots \text{Eq. (1)}$$

Also to accept lots of rejectable quality 5% of the time means that

$$\frac{K-RPL}{\sigma/\sqrt{n}} = +1.645 \qquad \ldots\ldots \text{Eq. (2)}$$

where APL = An Acceptable Process Level that yields product quality that we want to accept almost all $(1-\alpha)$ of the time it is offered, (this is usually the universe mean.)

RPL = A Rejectable Process Level that yields product quality that should be rejected almost all $(1-\beta)$ of the time it occurs.

K = Desired value for acceptance of the lot.

Subtracting (1) from (2) gives

$$\text{APL} - \text{RPL} = \frac{(2.054 + 1.645)\sigma}{\sqrt{n}}$$

for $n=5$, $\text{APL} - \text{RPL} = -1.65\,\sigma$

The acceptance tolerance E can now be calculated from the equation:

$$E = -\frac{t\sigma}{\sqrt{n}} = -\frac{2.054}{\sqrt{n}}\sigma = -.92\,\sigma$$

$$\text{and} \qquad K = \overline{X}' - .92\,\sigma$$

Knowing the value of the universe mean $\overline{X}'$ and the standard deviation for the variable, the value of K can be computed.

Example: For base course compaction with pooled $\sigma = 3.31$ and pooled mean = 99.28, the K value for acceptance would be $\overline{X}' - .92\,(3.31)$ or 96.23% and our plan would operate as follows:

1. Make 5 density determinations at locations randomly designated by the engineer.
2. Find $\overline{X}$, the mean per cent compaction of these five determinations.
3. If this mean is greater than or equal to 96.2%, accept the lot; otherwise, reject or take corrective action.

To see how this plan operates on lots of other means, an operating characteristic curve (OC) is constructed. From this curve (Fig. 9), the probability of accepting lots of various mean values can be determined.

Because of the mathematical relationship between sample size n and α and β , any change in n will produce considerable change in the risks.

TABLE IV

CALCULATION FOR OC CURVE FOR SAMPLING PLAN FOR BASE COMPACTION (ONE WAY PROTECTION)

n=5 K = 96.2

$\bar{X}$	$t = \dfrac{K - \bar{X}}{\sigma / \sqrt{n}}$	Probability of Acceptance P_{acc}
93.0	2.16	.0156
94.0	1.49	.0681
95.0	.81	.2090
96.0	.14	.4443
97.0	-.54	.7054
98.0	-1.22	.8888
99.0	-1.89	.9706
100.0	-2.97	.9949

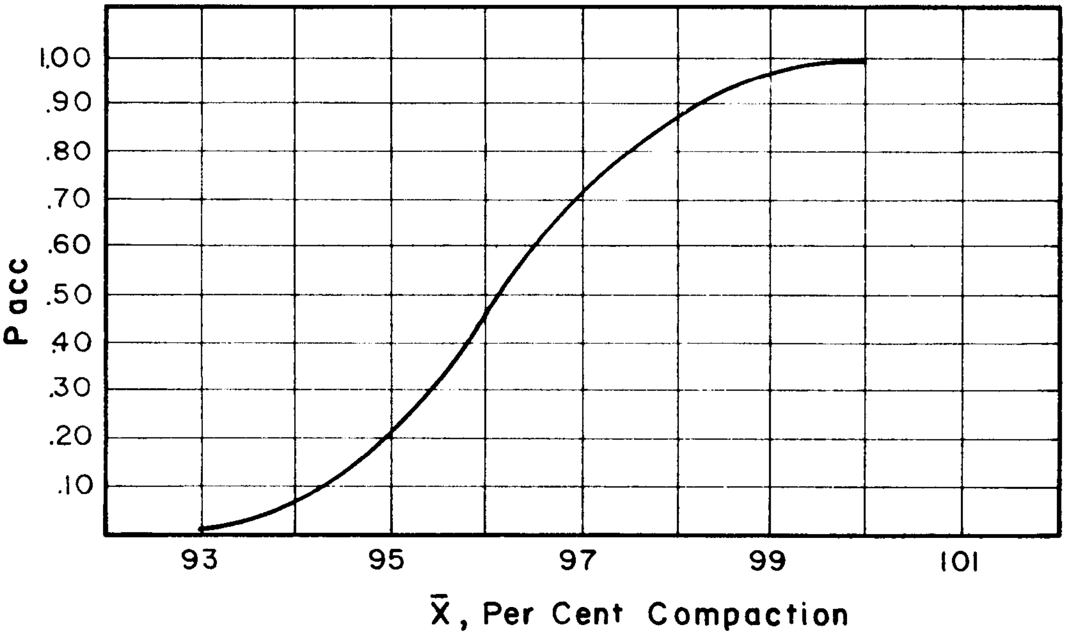


Figure 9: Operating Characteristic Curve (OC) for the Sampling Plan for Compaction of Base Course.

Two-Way Protection on Means

The above plan was for variable requiring only one-way protection. Similar reasoning can be applied to variables requiring two-way protection.

Again assuming normal distribution of sample means the problem can be illustrated graphically thus:

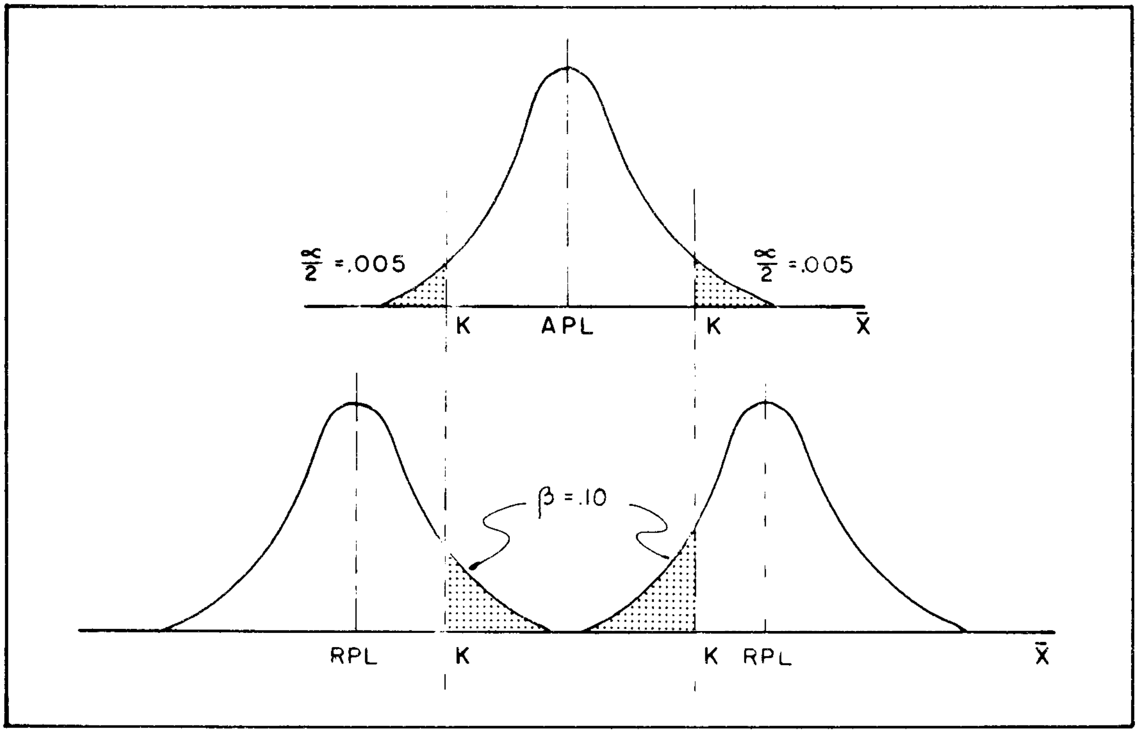


Figure 10: Distribution of $\bar{X}$ for Acceptable and Rejectable Process Levels and the Corresponding Risks (Two-way Protection on Means).

In this case, the value of α and β are as shown in the figure.

Here again, the choice on risks depends on the criticality of the characteristic. If not too critical (gradation of aggregate or depth of base course), then those indicated are satisfactory.

The equations needed to arrive at n and K are

$$\frac{K_1 - RPL}{\sigma / \sqrt{n}} = +1.282 \qquad \ldots\ldots \text{Eq. (1)}$$

$$\frac{K_2 - RPL}{\sigma / \sqrt{n}} = -1.282 \qquad \ldots\ldots \text{Eq. (2)}$$

$$\frac{K_1 - APL}{\sigma / \sqrt{n}} = -2.576 \qquad \ldots\ldots \text{Eq. (3)}$$

$$\frac{K_2 - APL}{\sigma / \sqrt{n}} = +2.576 \qquad \ldots\ldots \text{Eq. (4)}$$

where APL and RPL are the acceptable and rejectable process levels.

Solving these equations for $n=4$ will give

$$K_1 = \bar{X}' - 1.29\,\sigma$$

$$K_2 = \bar{X}' + 1.29\,\sigma$$

Example: For thickness of 6" soil cement base course

$$\sigma = .47$$

$$\bar{X}' = 6.25'' \qquad \text{(Table IX)}$$

Therefore, acceptance limits for mean of 4 depths should be

$$K_1 = \bar{X}' - 1.29\,\sigma = 5.64''$$

$$K_2 = \bar{X}' + 1.29\,\sigma = 6.86''$$

Thus, the aforementioned plan for two way protection for thickness of base course would operate as follows:

a. Make 4 depth measurements at randomly selected locations.

b. Find $\bar{X}$, the mean depth of these holes in inches.

c. If $\bar{X}$ is between 5.64" and 6.86" , accept the lot as conforming to the specifications; otherwise, reject or take corrective action.

The effectiveness of this sampling plan in accepting good lots and rejecting poor ones is illustrated by the OC curve (Figure 11). Table V gives the necessary calculations for points on the curve.

TABLE V

CALCULATIONS FOR OC CURVE FOR SAMPLING PLAN FOR THICKNESS OF BASE COURSE (TWO-WAY PROTECTION)

n=4 K_1 = 5.64" K_2 = 6.86"

$\bar{X}$	$t_1 = \frac{K_1 - \bar{X}}{.47/\sqrt{4}}$	$t_2 = \frac{K_2 - \bar{X}}{.47/\sqrt{4}}$	Probability of Acceptance P_{acc}
5.25	+1.66	6.85	.0485
5.45	+ .81	6.00	.2090
5.65	- .04	5.15	.5159
5.85	- .89	4.30	.8133
6.05	-1.74	3.45	.9591
6.25	-2.60	2.60	.9906
6.45	-3.45	1.74	.9591
6.65	-4.30	.89	.8133
6.85	-5.15	.04	.5159
7.05	-6.00	-.81	.2090
7.25	-6.85	1.66	.0485

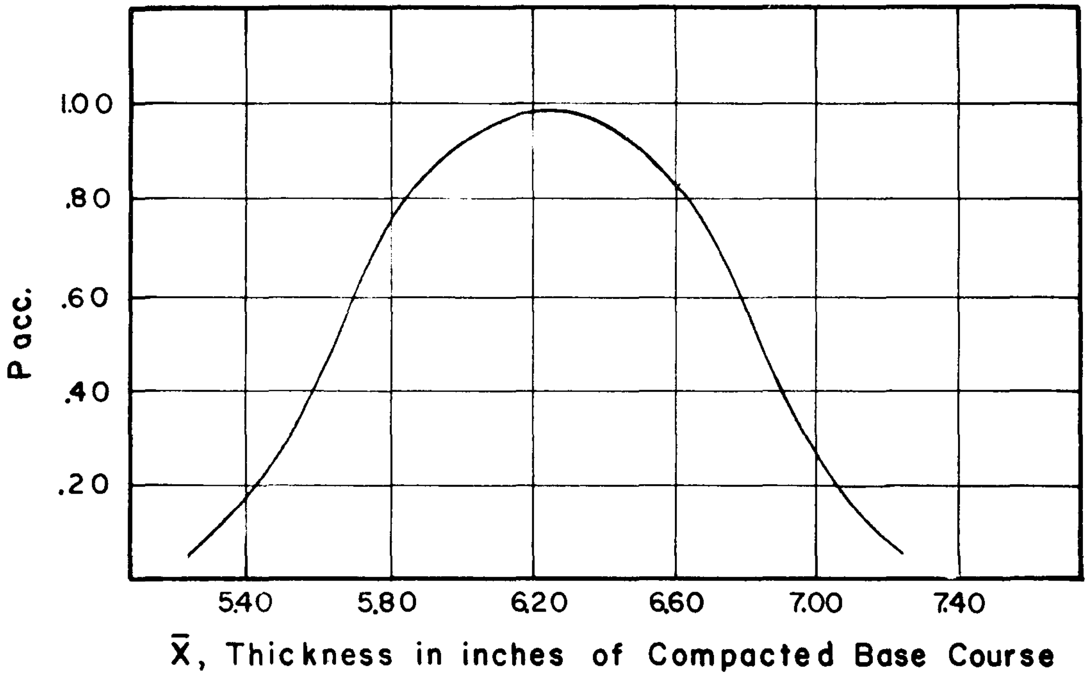


Figure 11: Operating Characteristic Curve for the Sampling Plan for Thickness of Base Course.

Protection on Individuals

A major disadvantage in using acceptance sampling for means of variables is that the mean of the sample may conform to some set value regardless of the individual values in the sample which may be too low and the rest sufficiently high to give the desired average. To safeguard against this, limits on individuals should also be enforced along with those on the means.

In the preceding section where the size of the sample has already been established for certain risks; it is only necessary to use criteria which can tell whether any single measurement from the sample could be considered to be from population other than that sampled. This criteria for outliers is explained in ASTM E 178. Reference (4) treats this subject in more detail.

The following limits are those suggested for selected α's:

α	Limits	Sample Size
.02	$\bar{x}_s \pm 2.48\sigma$	5
.01	$\bar{x}_s \pm 2.62\sigma$	4

where $\bar{x}_s$ is the average of the number of measurements in the sample.

Table VI summarizes the sampling plan described in the preceding section. The sample size n indicated is not a limiting value but a definite one and any change in the number will affect the corresponding risks and hence the tolerances.

TABLE VI

SUMMARY OF STATISTICAL ACCEPTANCE SAMPLING FOR BASE COURSE CHARACTERISTICS

Material Characteristic	Base Course	P_a	P_r	Sample Size n	Acceptance Limits				Frequency of Measurement
					Mean		Individuals		
					LL	UL	$\bar{x}_s-$	$\bar{x}_s+$	
% Compaction	Stab.	98	95	5	96. 2	-	8. 20	-	One every 1000 sq. yds.
% Compaction	Raw	98	95	5	100. 8	-	7. 40	-	
Thickness									One every 1000 sq. yds.
6"	Stab.	99	90	4	5. 64	6. 86	1. 23	1. 23	
8"	Stab.	99	90	4	7. 56	9. 04	1. 49	1. 49	
8. 5"	Stab.	99	90	4	8. 04	9. 40	1. 39	1. 39	
10"	Raw	99	90	4	9. 00	11. 40	2. 44	2. 44	
Gradation	SCG"A"								One every 500 cu. yds. in place
% Passing 3/4"		99	90	4	86. 54	93. 06	6. 63	6. 63	
No. 4		99	90	4	49. 70	61. 38	11. 87	11. 87	
No.40		99	90	4	26. 01	39. 35	13. 55	13. 55	
No.200		99	90	4	10. 93	17. 41	6. 58	6. 58	
Gradation	SCG"B"								One every 500 cu. yds. in place
% Passing									
No.4		99	90	4	60. 02	73. 44	13. 62	13. 62	
No. 40		99	90	4	34. 73	48. 67	14. 15	14. 15	
No. 200		99	90	4	10. 93	17. 41	10. 61	10. 61	
Moisture Content									
Soil Cement	Stab.	99	90	4	-2. 40*	+2. 40*	-	-	One every 1000 sq. yds.
Width									One every 1000 sq. yds.
if < 22'	Stab.	99	90	4	-. 35**	+. 35**	-	-	
if > 22'	Stab.	99	90	4	-. 54**	+. 54**	-	-	

* The limits are to be calculated on the basis of recommended optimum moisture content.

** The limits are to be calculated on the basis of plan width.

SUMMARY

In the preceding sections, an attempt has been made to determine the extent of variability on base course characteristics using data collected from completed project files. On the basis of this variability, numerical limits have been established using statistical quality control technique. The analysis can be summed up in the following statements:

(1) Frequency distribution of the historical data for most of the base course characteristics tend to follow normal distribution.

(2) The variability for base course compaction as expressed by the standard deviation σ is considerably different for different Contractors (Projects). This σ is however more pronounced for cement stabilized aggregate base course than for stabilized soil cement base course.

(3) For raw or unstabilized aggregate base course, the variability is less than that for stabilized base course.

(4) The size of the sample and the acceptance limits for the mean of the sample are definite values and any attempt to improvise would necessarily change the probability of acceptance and rejection.

RECOMMENDATIONS

The effectiveness of the sampling plan discussed in the preceding section and summarized in Table VI can only be evaluated when applied to actual jobs during construction. Therefore, it is suggested that three separate projects be selected in different parts of the State for such an evaluation. The sampling plan for means and individuals and the corresponding acceptance limits will have to be incorporated in the contract specification under special provisions.

Another point that needs to be mentioned is the use of non-destructive test method for density determinations. Although Louisiana Department of Highways has approximately 25 such units, its use has been still experimental. Conventional methods are still used for job control but it would not be long before they will be replaced by nuclear equipment, since these have been well accepted as regards their reliability, adaptability, and precision. Regardless of when this is done, the acceptance specifications will have to be re-evaluated on the basis of this new equipment. It is therefore recommended that the Louisiana Department of Highways give consideration to the following method for field evaluation of the nuclear equipment.

The method, or rather technique, can be called the Control Strip Technique and basically involves the construction of a control strip on the roadway prior to construction of the job. This is accomplished by repeated rolling of the base course with specified rolling equipment until no increase in density can be detected. The moisture content is kept at the optimum that will give the maximum density with the available man, machinery, and material. The final density obtained in the control strip then, serves as the basis for controlling the remaining part of the project in sections. All the tests are made with nuclear equipment.

Statistical tools can be applied to answer some of the questions pertaining to:

1. The length of the control strip.
2. Number of test sites in control strip.
3. Number of readings per test site in control strip.
4. Length of each test section, etc.

To answer the above questions, knowledge will have to be gained of the variability of the characteristic using the nuclear equipment. Probably the best approach would be to design an experiment for ANOVA (Analysis of Variance).

Using the aforementioned technique will provide field compaction testing with a minimum of time and personnel and will definitely prove to be superior to the conventional method from the stand point of quality control.

Adoption of statistical techniques for determining job compliance will undoubtedly present multitude of factors, some large involving administrative decisions and others small enough to be tackled by field personnel. Any attempt to list some of the major implications involved in implementation of such a program would be a report in itself. However, one such deserves some space here.

Unfortunately, lack of specific statistical background can be considered a major deterent to the over-all program. In the industry, the producer and the consumer are blessed with the services of quality control engineers and generally no questions are raised whenever the product fails to meet the preset specification requirement. In the highway industry, eyebrows will undoubtedly be raised. To convince them would require setting up of an educational program and training courses in statistical quality control techniques for senior inspectors and project engineers. The producers' group should be particularly encouraged to participate in such a program.

The above program or course should be simple and given in terms that are easily understood limiting the mathematics to an absolute minimum. It should be basic. Only such topics as frequency distribution, measures of variability, control charts, and acceptance sampling by variables should be covered. Once these basic principles are grasped, actual application to different highway materials (soils, concrete, etc.) can be covered which would further enhance self interest for the personnel concerned. Only then will it be clear to the parties concerned (particularly the contractor) why the limits on means are more stringent than those on individuals, the importance of the number of measurements or size of sample, or the significance of a point falling outside the control limits on the Control Chart.

Summing up this report, the correctness of any specification is determined by the mathematical relationship between use and production coupled with cost. Specifications fundamentally should be based on facts and these must be known first. They must come from a study of the product, of its use, and its production. Last but not least, the correct specifications mean an open, cooperative effort by the producer and consumer. Each must be conversant with the problems of the other and both must be willing to study the over-all problem.

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APPENDIX

TABLE VII

SUMMARY OF STATISTICAL RESULTS ON STABILIZED BASE COURSE COMPACTION

Project Ident.	No. of Obs. n	Mean $\bar{X}'$	Variance σ^2	Std. Dev. σ	Min.	Max.	% Outside Specification
Stabilized Soil Cement Base Course							
A	1000	98.71	3.84	1.96	88.6	104.8	3.6
B	736	98.98	8.83	2.97	89.2	116.8	2.6
C	630	97.69	7.48	2.73	86.1	105.4	10.0
D	615	99.11	8.98	3.00	85.2	108.2	5.2
E	527	98.94	9.98	3.16	90.8	104.2	4.4
F	290	99.90	10.82	3.29	89.3	110.0	5.5
G	280	99.43	10.20	3.19	90.4	107.9	2.1
H	265	98.40	9.19	3.03	88.3	105.7	7.9
I	110	98.63	15.86	3.98	86.3	115.2	6.1
Stabilized Sand - Clay Gravel Base Course							
J	468	102.74	26.67	5.16	90.0	127.4	2.8
K	134	99.97	16.85	4.11	91.0	111.4	5.2
Stabilized Sand Shell Base Course							
M	385	100.49	22.68	4.75	83.0	114.3	9.4
Pooled Values of the Above							
	5440	99.28	10.93	3.31			

TABLE VIII

SUMMARY OF STATISTICAL RESULTS ON UNSTABILIZED (RAW) BASE COURSE COMPACTION

Project Ident.	No. of Obs. n	Mean $\bar{X}'$	Variance σ^2	Std. Dev. σ	Min.	Max.	% Outside Specification
Sand - Clay Gravel Base Course							
A	210	104.73	12.28	3.50	99.9	114.6	2.4
B	177	103.01	4.96	2.23	100.0	109.1	0
Sand Shell Base Course							
C	96	102.29	9.43	3.07	96.0	111.0	5.2
D	73	102.98	5.32	2.31	100.0	109.1	1.4
Pooled Values of the Above							
	556	103.53	8.66	2.93			

TABLE IX

SUMMARY OF STATISTICAL RESULTS ON STABILIZED BASE COURSE THICKNESS

Project Ident.	No. of Obs. n	Mean $\overline{X}'$	Variance σ^2	Std. Dev. σ	Min.	Max.	% Outside Specification	
							below	above
Stabilized Soil Cement Base Course - 6" Thickness								
A	292	6.14	.18	.42	5.00	7.50	3.40	1.40
B	207	6.25	.30	.55	5.00	9.00	1.00	2.90
C	162	6.60	.19	.42	5.50	7.75	0	8.00
D	143	5.84	.15	.39	5.00	7.50	.70	.70
E	141	6.51	.27	.52	5.00	8.00	4.30	4.30
	Pooled Values of the Above							
	946	6.25	.22	.47				
Stabilized Soil Cement Base Course - 8" Thickness								
F	272	8.61	.42	.65	6.00	11.30	.70	11.80
	Stabilized Sand - Clay Gravel Base Course							
G	134	7.92	.28	.53	7.00	9.30	11.90	.70
H	100	8.10	.16	.40	7.00	9.30	1.00	0
	Unstabilized (raw) Sand - Clay Gravel Base Course							
I	77	8.14	.32	.56	7.50	10.80	0	3.90
	Pooled Values of the Above							
	583	8.30	.33	.57				

TABLE X

SUMMARY OF STATISTICAL RESULTS ON GRADATION OR GRADE "A" SAND-CLAY-GRAVEL FOR BASE COURSE

Project Ident.	% Passing U. S. Sieve	No. of obs. n	Mean $\overline{X}'$	Variance σ^2	Std. Rev. σ	Min.	Max.	% Off Specifications	
								below	above
A	3/4"	257	90.53	5.48	2.33	83	97	0	1.20
	No. 4	257	54.78	24.06	4.90	43	72	0	9.30
	No. 40	257	36.79	40.27	6.33	17	60	.40	17.50
	No. 200	257	16.50	8.29	2.90	5	24	1.90	8.60
B	3/4"	352	89.32	7.13	2.70	73	98	.30	.90
	No. 4	352	56.10	18.16	4.30	45	68	0	13.40
	No. 40	352	29.67	16.83	4.80	21	45	0	0
	No. 200	352	12.47	4.86	2.20	7	21	6.30	.30
	Pooled Values of the Above								
	3/4"	609	89.80	6.43	2.53				
	No. 4	609	55.54	20.65	4.53				
	No. 40	609	32.68	26.72	5.17				
	No. 200	609	14.17	6.31	2.51				

TABLE XI

SUMMARY OF STATISTICAL RESULTS ON GRADATION OF GRADE "B" SAND-CLAY-GRAVEL FOR BASE COURSE

Project Ident	% Passing U. S. Sieve	No. of Obs.	Mean $\bar{X}'$	Variance σ^2	Std. Dev. σ	Min.	Max.	% Off Specifications	
								below	above
A	No. 4	790	66.64	28.07	5.30	53	82	0	5.70
	No. 40	790	40.94	30.92	5.56	15	59	.3	5.80
	No. 200	790	17.48	18.69	4.32	4	40	4.90	4.10
B	No. 4	123	71.59	17.03	4.11	60	85	0	17.90
	No. 40	123	42.51	19.62	4.43	35	55	0	5.70
	No. 200	123	13.03	8.41	2.90	6	21	36.60	0
C	No. 4	88	60.76	35.43	5.95	48	73	1.10	0
	No. 40	88	47.36	21.87	4.68	33	65	0	22.70
	No. 200	88	17.02	6.39	2.52	11	32	1.10	1.10
	Pooled Values of the Above								
	No. 4	1001	66.73	27.36	5.20				
	No. 40	1001	41.70	28.78	5.40				
	No. 200	1001	16.82	16.40	4.05				

TABLE XII

SUMMARY OF STATISTICAL RESULTS ON BASE COURSE CHARACTERISTICS (CONTROLLED FIELD EXPERIMENT)

Project Identification	No. of Obs. n	Mean $\bar{X}'$	Variance σ^2	Std. Dev. σ	Min	Max
Base Course Compaction, %						
A	30	94.80	17.00	4.12	87.6	102.8
B	28	97.05	10.98	3.31	87.0	102.6
C	27	95.46	8.21	2.87	89.9	101.5
D	19	97.66	21.77	4.64	86.7	103.2
Depth of Base Course*, in.						
B	84	8.51	.20	.45	7.50	9.25
C	81	9.02	.29	.54	7.75	10.00
	(81)	(8.79)	(.49)	(.70)	(7.00)	(11.00)
D	55	8.60	.41	.64	7.50	10.25
	(52)	(9.75)	(2.22)	(1.50)	(7.75)	(13.75)
Moisture Content, %						
A	100	11.68	2.88	1.70	8.1	17.0
B	56	10.92	1.94	1.40	8.4	13.6
C	96	15.15	4.15	2.04	10.0	20.6
D	74	14.51	4.22	2.05	10.6	18.3
Width of Base Course, ft						
B	28	21.24	.081	.28	20.75	21.67
C	26	21.53	.158	.40	26.00	27.70
D	19	22.39	.066	.26	22.08	23.08

* Augered depth
( ) Stringline depth

GLOSSARY

APL - An Acceptable Process Level that yields product quality that should be accepted almost all of the time.

E - The specification tolerance which determines acceptance limits.

K - The desired value for acceptance of the lot.

LSL - Lower Specification Limit.

LL - Lower Limit.

N - Number of observations in a group or subgroup.

P_a - Probability of accepting good material having the desired average value.

P_r - Probability of rejecting bad material having the lowest acceptable average.

RPL - A Rejectable Process Level that yields product quality that should be rejected almost all of the time.

t - The normal deviate or the number of standard deviations of the measured characteristic above or below the mean value as measured on the horizontal axis.

USL - Upper Specification Limit.

UL - Upper Limit.

X_i - The value of a single measured characteristic.

$\bar{X}$ - The average or arithmetic mean found by dividing the sum of n observations by the number of observations.

$\bar{X}_s$ - The average of a number of measurements in a sample.

$\bar{X}'$ - The universe mean or the true average quality of the measured characteristic.

σ - (Sigma) - The Standard deviation which is a measure of the dispersion of a group of measurements from their average.

α - (Alpha) - The Producer's risk or the probability of rejecting lots of acceptable mean quality.

β - *(Beta) - The Consumer's risk or the probability of accepting* lots of rejectable mean quality.

Σ - A symbol for summation of values.

$<$ - A symbol for "less than".

$>$ - A symbol for "greater than".

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